

Final is Saturday, December 9th

KANE 130, 1:30-4:20pm

8 page final, comprehensive.

Allowed:

- One 8.5 by 11 inch sheet of handwritten notes (front and back).
- A Ti-30x IIS Calculator

Entry Task: Course Evaluation

Get out your computer/smart phone, go here, and fill out the evaluation:

125B (10:30 Lecture):

uw.iasystem.org/survey/181783

125C (11:30 Lecture):

uw.iasystem.org/survey/181781

Evaluation Notes

- This eval. is for me and the lecture/class (TA will have a different eval. for quiz section).
- I will not see the results until next quarter (I will never see your name)
- The comments only go to me.

Course best described as...:

“In your major” means you are a math major.

For the vast majority of you, this course is a “core/distribution requirement”.

EXAM 1 MATERIAL

Riemann Sums, integral def'n,
Fund. Thm of Calc (Both Parts),
Net Change and Total Change,
Substitution, Areas between Curves,
Volumes of solids.

EXAM 2 MATERIAL

Work, Average Value,
Integration by parts, Partial Fractions,
Trig Integrals, Trig Substitution,
Approximating Integrals, Improper.

NEW MATERIAL

Arc Length
Center of Mass
Solving separable differential.
Solving and interpreting a differential
equation in a “story” problem.

1. (10 total points) Evaluate the following indefinite integrals.

(a) (5 points) $\int t^5 \sin(t^3) dt$

(b) (5 points) $\int \frac{1}{x(x+\sqrt{x})} dx$

2. (10 total points) Evaluate the following definite integrals.

(a) (5 points) $\int_0^1 \ln(1+t^2) dt$

(b) (5 points) $\int_0^2 \frac{x^3}{\sqrt{4+x^2}} dx$

1. Evaluate the following indefinite integrals.

(a) (5 points) $\int \sin(x) \sqrt{\cos(x)} dx$

(b) (5 points) $\int \sqrt{3 - 2x - x^2} dx$

2. Evaluate the following definite integrals.

(a) (5 points) $\int_0^{\pi} \sec\left(\frac{x}{3}\right) \cdot \tan^3\left(\frac{x}{3}\right) dx$

(b) (5 points) $\int_{-1}^2 \frac{x}{x^2 + 2x + 10} dx$

7. Let \mathcal{R} be the region bounded by the curve $y = \sqrt{x}$, the line $x = 4$, and the x -axis.

(a) (6 points) The region \mathcal{R} is rotated around the line $x = 6$ to form a solid. Set up an integral for the volume of this solid using cylindrical shells and **EVALUATE THE INTEGRAL**.

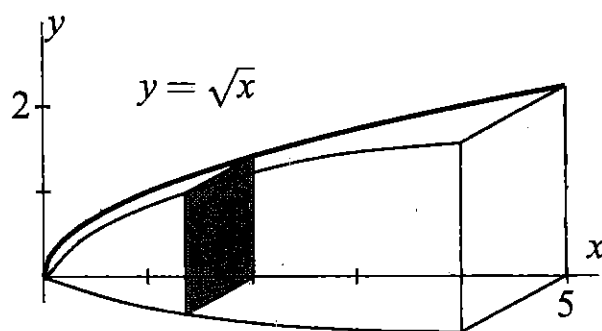
(b) (4 points) Set up an integral for the volume of this solid using washers. **DO NOT EVALUATE THE INTEGRAL**.

3. (10 total points) A balloon is moving vertically up and down along a straight line above the ground, with the positive direction pointing up. The acceleration of the balloon at time t (in seconds) is given by $a(t) = -(t + 5)$ ft/sec². The initial velocity of the balloon at time $t = 0$ is $v(0) = 12$ ft/sec.
- (a) (3 points) Find the velocity $v(t)$ of the balloon as a function of time t .

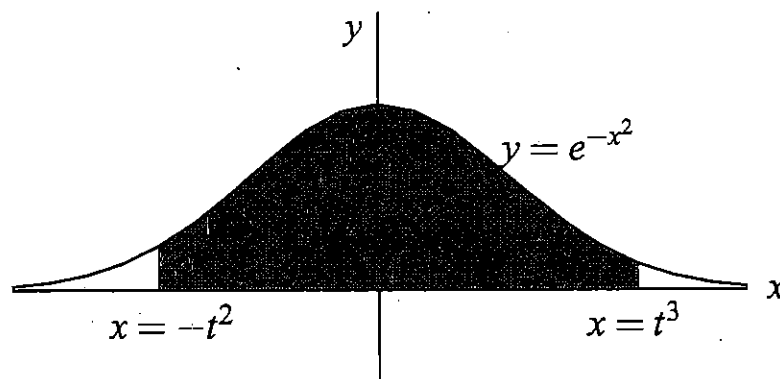
(b) (4 points) Find the *total distance* traveled by the balloon from time $t = 0$ sec to time $t = 3$ sec.

(c) (3 points) The balloon hits the ground at time $t = 6$ sec. What was its initial height above the ground at time $t = 0$?

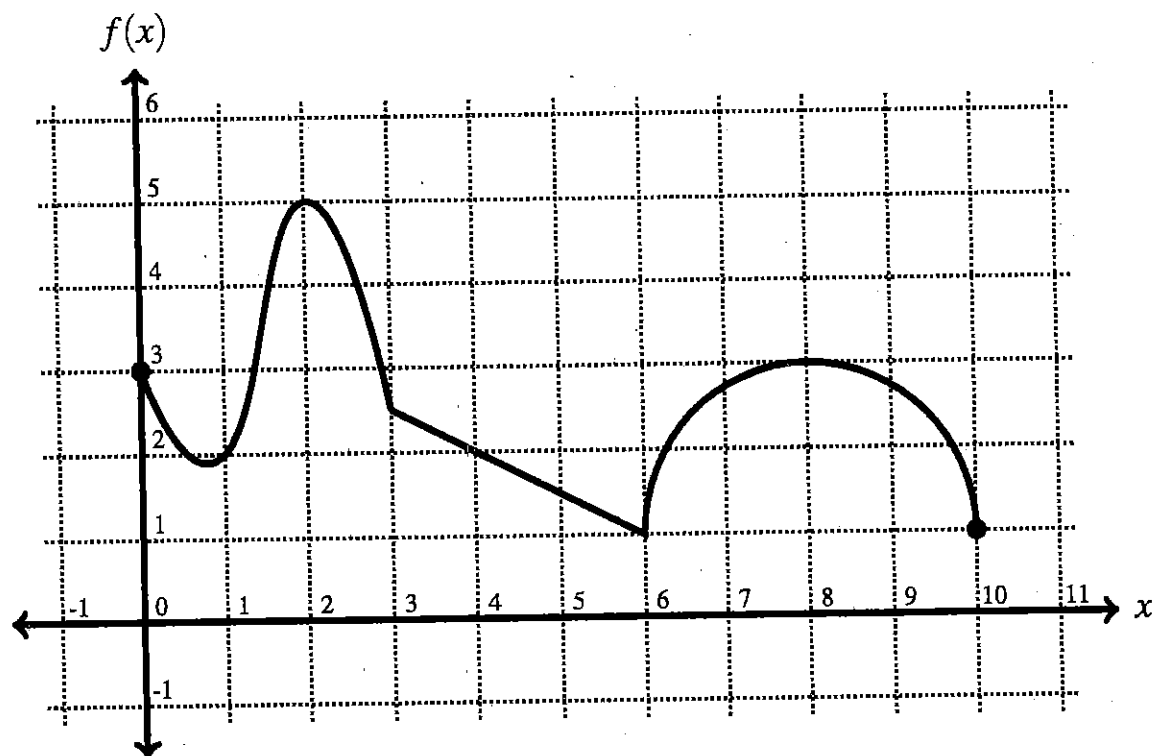
5. (4 points) Find the volume of the solid shown below. Each cross-section (slice) is a square.



6. (8 points) At each time $t \geq 0$, \mathcal{R}_t is the region above the x-axis, below the curve $y = e^{-x^2}$, with left side on the line $x = -t^2$ and right side on the line $x = t^3$ (see the figure). Let $A(t)$ be the area of \mathcal{R}_t . Find $\frac{dA}{dt}$ at time $t = 1$.



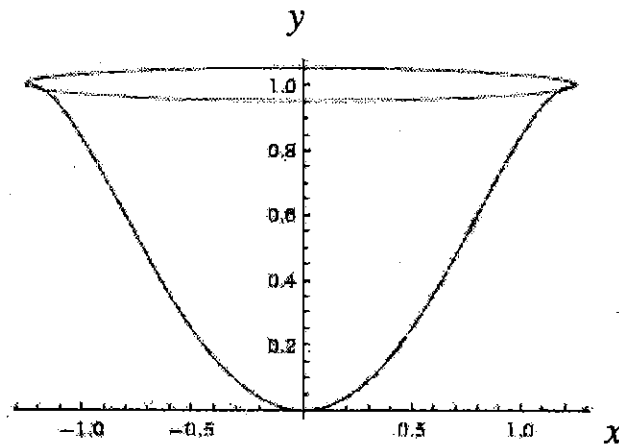
8. The graph of $f(x)$ is shown below. Use it to answer the following questions.



(a) (4 points) Compute the average value of $f(x)$ on the interval $[4, 10]$.

(b) (6 points) Let $g(x) = \int_{x^2}^7 f(t) dt$. Calculate $g''(2)$.

6. (10 total points) The curve $x = \sqrt{\sin^{-1} y}$ for $0 \leq y \leq 1$ is rotated around the y -axis to form a container. The container is filled with a fluid that weighs 40 lb/ft^3 . Length units for x and y are in feet.

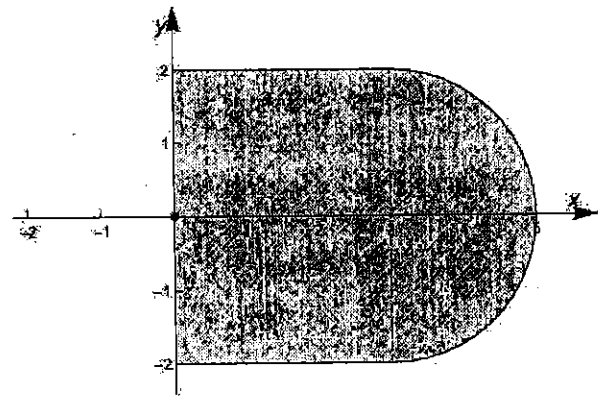


- (a) (6 points) Set up a definite integral (with respect to y) for the work (in ft-lb) required to empty the container by pumping all of the fluid to the top of the container.
(Note: Do not use the acceleration due to gravity; pounds are already a unit of force.)
IN THIS PART, DO NOT EVALUATE THE INTEGRAL YET.

- (b) (4 points) Now evaluate the integral in part (a). Give your answer in exact form.

6. (10 points) An 80-ft cable is used to lift 50 pounds of coal up a mine shaft 80 ft deep. The bottom half of the cable weighs 2 pounds per foot and the top half of the cable weighs 3 pounds per foot. Find the work done in foot-pounds.

3. (10 points) Consider the region in the xy -plane formed by a rectangle of height 4 and width 3 and a half-disk of radius 2 centered at $(3, 0)$, as shown in the figure. Compute \bar{x} , the x -component of the centroid of the region.



9. (10 points) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{(x+3)(y+2)}{x^2+9}, \quad y(0) = 10.$$

Give your answer in the form $y = f(x)$.